7. Spline Functions Based on IAPWS-95

The IAPWS-95 formulation for general and scientific use [4, 5] is the most accurate representation of the thermodynamic properties of water and steam. The IAPWS-IF97 formulation for industrial use [2, 3] and the supplementary releases [6, 7, 8, 9] were developed based on IAPWS-95 to meet specific needs for higher computing speeds in many industrial applications, particularly for the steam power industry. The range of validity of IAPWS-IF97 is divided into five regions, resulting in small inconsistencies at the region boundaries. In situations where these inconsistencies cannot be tolerated, and/or for general and scientific use where the more accurate IAPWS-95 formulation is preferred, it may be useful to apply similar spline techniques to IAPWS-95. In order to demonstrate the applicability of the SBTL method to IAPWS-95, several spline-based property functions for calculations from (v,u) and (p,h) have been developed. For simplicity in developing this example, spline functions covering the region of temperatures from 273.15 K to 1273.15 K and pressures up to 1000 MPa are described. This excludes a small portion of the range of validity of IAPWS-95 at high pressures and low temperatures, but application of the SBTL method in that region would be a straightforward extension.

7.1. Spline Functions of (*v*,*u*)

Spline functions based on IAPWS-95 for the calculation of $p, T, s, w = f(v, u)$ in the single-phase region were created analogously to those based on IAPWS-IF97 (see Sec. 4). The results of the computing-time comparisons are summarized in Sec. 7.3.

7.1.1. Range of Validity

The range of validity covers the fluid range of state bounded as follows:

$$273.15\ \text{K} \le T \le 1273.15\ \text{K} \qquad 611.212\ \text{Pa} \le p \le 1000\ \text{MPa}\,.$$

This range of validity corresponds to that of IAPWS-95, except for the lower temperature limit, which is 273.15 K, and the lower pressure limit, which is $p_\text{s}(273.15\ \text{K}) = 611.212\ \text{Pa}$. Figure 12 shows the range of validity and the defined regions of the spline functions with the variables (v,u). The range of validity is divided into the liquid region L, the gas region G, and the two-phase region TP. This division is similar to the division for IAPWS-IF97 shown in Fig. 9, except that no separate high-temperature region HT is needed.

The specific internal energy at the critical point $u_\text{c} = 2015.734\,524$ kJ/kg is used to define the boundary between regions L and G for supercritical state points. At the region boundary in the single-phase region, small inconsistencies are unavoidable. These should be negligible for most purposes, but if needed the transition at this boundary can be smoothed using simple interpolation equations.

in the gas phase derived from $p^{\mathrm{G}}(v,u)$. The iteration procedure is repeated until $|f| \leq TOL$ and $p_{\mathrm{s}} = p_k$, T_{s}, x, v', v'', u', u'', s', and s'' are determined. A spline function for $p_{\mathrm{s}}(u,s)$ is used to initialize p_k.

A7 Transformations and Grid Dimensions

For each spline function described in Secs. 4, 5, 6, and 7, the transformations and dimensions of the grid of nodes are given in the tables below. For piecewise equidistant nodes, the domain of the considered transformed variable $\overline{x}_{\min} \leq \overline{x} \leq \overline{x}_{\max}$ is subdivided in several intervals with equidistant nodes. In the tables below, this is described with

$$\begin{bmatrix} \overline{x}_{\min} \\ \ldots \\ \ldots \\ \overline{x}_{\max} \end{bmatrix} \begin{bmatrix} \text{nodes} \\ \ldots \\ \text{nodes} \end{bmatrix},$$

where the boundaries of the intervals are given in the column on the left and the number of equidistant nodes between them is given in the column on the right.

Table A1: Transformations and dimensions of the grid of nodes of each (v,u) spline function for the liquid region L based on IAPWS-IF97 and the IAPWS viscosity release with recommendations for industrial use [13]

Spline function	v [m^3/kg]	u [kJ/kg]
	$\bar{v}(v,u)=\dfrac{\bar{v}_{\max}-\bar{v}_{\min}}{v_{\max}(u)-v_{\min}(u)}\cdot\left(v-v_{\min}(u)\right)+\bar{v}_{\min}$ $\bar{v}_{\min}=1 \qquad \bar{v}_{\max}=100$	
	$\begin{bmatrix}\bar{v}_{\min}\\ \ldots\\ \ldots\\ \bar{v}_{\max}\end{bmatrix}\begin{bmatrix}\text{nodes}\\ \ldots\\ \text{nodes}\end{bmatrix}$	$\begin{bmatrix}u_{\min}\\ \ldots\\ \ldots\\ u_{\max}\end{bmatrix}\begin{bmatrix}\text{nodes}\\ \ldots\\ \text{nodes}\end{bmatrix}$
$p^{\mathrm{L}}(v,u)$	$\begin{bmatrix}1\\ 95\\ 100\end{bmatrix}\begin{bmatrix}100\\ 200\end{bmatrix}$	$\begin{bmatrix}-8.489\,68\\ 250\\ 2040.01\end{bmatrix}\begin{bmatrix}300\\ 225\end{bmatrix}$
$T^{\mathrm{L}}(v,u)$	$\begin{bmatrix}1\\ 100\end{bmatrix}\begin{bmatrix}100\end{bmatrix}$	$\begin{bmatrix}-8.489\,68\\ 2040.01\end{bmatrix}\begin{bmatrix}200\end{bmatrix}$
$s^{\mathrm{L}}(v,u)$	$\begin{bmatrix}1\\ 100\end{bmatrix}\begin{bmatrix}100\end{bmatrix}$	$\begin{bmatrix}-8.489\,68\\ 10\\ 2040.01\end{bmatrix}\begin{bmatrix}10\\ 200\end{bmatrix}$
$w^{\mathrm{L}}(v,u)$	$\begin{bmatrix}1\\ 90\\ 100\end{bmatrix}\begin{bmatrix}100\\ 50\end{bmatrix}$	$\begin{bmatrix}-8.489\,68\\ 10\\ 2040.01\end{bmatrix}\begin{bmatrix}10\\ 200\end{bmatrix}$
$\eta^{\mathrm{L}}(v,u)$	$\begin{bmatrix}1\\ 100\end{bmatrix}\begin{bmatrix}100\end{bmatrix}$	$\begin{bmatrix}-8.489\,68\\ 300\\ 2040.01\end{bmatrix}\begin{bmatrix}75\\ 150\end{bmatrix}$

Table A10: Transformations and dimensions of the grid of nodes of each (p,h) spline function for the liquid region L and the gas region G based on IAPWS-95

Spline function	p [MPa]		h [kJ/kg]
	$\bar{p}(p)$	$\begin{bmatrix} \bar{p}_{min} \\ \ldots \\ \ldots \\ \bar{p}_{max} \end{bmatrix} \begin{bmatrix} \text{nodes} \\ \ldots \\ \text{nodes} \end{bmatrix}$	$\begin{bmatrix} h_{min} \\ \ldots \\ \ldots \\ h_{max} \end{bmatrix} \begin{bmatrix} \text{nodes} \\ \ldots \\ \text{nodes} \end{bmatrix}$
$T^{L}(p,h)$	p	$\begin{bmatrix} 4.84693\times10^{-4} \\ 1\times10^{-2} \\ 20 \\ 100 \\ 1100 \end{bmatrix} \begin{bmatrix} 100 \\ 75 \\ 100 \\ 125 \end{bmatrix}$	$\begin{bmatrix} -13.3533 \\ 2140 \end{bmatrix} [150]$
$T^{G}(p,h)$	p	$\begin{bmatrix} 5\times10^{-4} \\ 3\times10^{-2} \\ 0.5 \\ 20 \\ 120 \\ 1100 \end{bmatrix} \begin{bmatrix} 150 \\ 75 \\ 100 \\ 100 \\ 150 \end{bmatrix}$	$\begin{bmatrix} 2040 \\ 2850 \\ 4679.71 \end{bmatrix} \begin{bmatrix} 75 \\ 100 \end{bmatrix}$
$v^{L}(p,h)$	p	$\begin{bmatrix} 4.84693\times10^{-4} \\ 1\times10^{-2} \\ 20 \\ 100 \\ 1100 \end{bmatrix} \begin{bmatrix} 100 \\ 75 \\ 100 \\ 125 \end{bmatrix}$	$\begin{bmatrix} -13.3533 \\ 2140 \end{bmatrix} [150]$
$v^{G}(p,h) = \dfrac{\bar{v}^{G}(p,h)}{p}$	p	$\begin{bmatrix} 5\times10^{-4} \\ 3\times10^{-2} \\ 0.5 \\ 20 \\ 120 \\ 1100 \end{bmatrix} \begin{bmatrix} 125 \\ 50 \\ 50 \\ 85 \\ 90 \end{bmatrix}$	$\begin{bmatrix} 2040 \\ 2850 \\ 4679.71 \end{bmatrix} \begin{bmatrix} 75 \\ 75 \end{bmatrix}$